Differential Algebraic Geometry
April 16-18
CUNY Graduate Center and NYU Courant Institute
Program

Friday, April 16:

Room 5382, Graduate Center, 365 5th Avenue

9:15 - 11 am	Sergey Gorchinskiy	Geometry and differential Tannakian categories
11 am – 12 pm	Sergey Gorchinskiy	Informal discussions on the above topic
	Lunch	
2 – 3:30 pm	Michael Singer	Differential groups and factorization of partial differential operators
3:45 – 5:15 pm	Camilo Sanabria	Reductive groups and ruled surfaces

Saturday, April 17:

Room 6417, Graduate Center, 365 5th Avenue

10:10 – 11:25 am	Henri Gillet	Derivations and descent in Characteristic p
11:35 am – 12:50 pm	Moshe Kamensky	Differential tensor categories and model theory
	Lunch	
2:15 – 3:45 pm	Michael, Moshe, Sergey, Henri	Open Problems
4 – 5:15 pm	Fedor Bogomolov	Closed holomorphic differentials and holomorphic webs

Sunday, April 18:

Room 201, Courant Institute, 251 Mercer Street

10 – 11:15 am	Ray Hoobler	Applying Picard-Vessiot theory to Brauer groups
11:30 am – 12:45 pm	Dmitry Trushin	Difference Nullstellensatz
	Lunch	
2:15 – 3:30 pm	Bruno de Oliveira	Geometry of closed symmetric differentials of rank 1 and 2

Abstracts of talks

Sergey Gorchinskiy
Steklov Institute, Moscow

Geometry and differential Tannakian categories

The talk is based on a common work with A. Ovchinnikov and H. Gillet. Parametrised differential Galois groups are symmetries of solution spaces of linear differential equations with parameters that commute with taking derivatives along the parameters. A general Galois theory of differential equations where Galois groups are differential algebraic groups was introduced by Landesman. The special case of parametrised linear differential equations was developed by Cassidy and Singer who discussed various special properties of the parameterized Picard-Vessiot Galois group when the field of parameters is differentially closed. We discuss a new approach to this based on differential Tannakian categories, Atiyah extensions, and some other constructions from algebraic geometry. As an application we obtain that in a wide range of cases, parametrised differential Galois groups can be defined over the field of parameters though this field is not differentially closed.

Michael Singer
North Carolina State University

Differential groups and factorization of partial differentia operators

An ordinary differential operator can be factored as a product of irreducible operators and any two such factorizations have the same number of factors and, after a possible permutation, these factors are equivalent in a suitable sense. Examples showing that such a result is not true for partial differential operators have been known for over 100 years.

Solutions of systems of homogeneous linear partial differential equations form a group under addition and are an example of a differential algebraic group. We show that a Jordan-Hoelder type theorem holds for such groups, that is, any such group can be filtered by a finite subnormal series of differential algebraic groups such that successive quotients are "almost simple". Furthermore, any two such series have the same length and, after a possible permutation, successive quotients are “isogenous”.

This talk will not assume that the audience is familiar with differential algebraic groups and will expose the necessary background. Special attention will be given to the meaning of this result in the context of systems of linear partial differential equations and many examples will be given.

Camilo Sanabria
CUNY Graduate Center

Reductive connections and Ruled surfaces

We consider a meromorphic connection with reductive Galois group over a compact connected Riemann Surface. In this setting, we take the projective space bundle defined by the symmetric algebra of rational horizontal sections. Using a result of E. Compoint and of M. Singer we prove that this projective space bundle is a ruled surface that characterizes the class of projectively equivalent connections. In particular different projectively equivalent connections correspond to different divisors of this ruled surfaces.

Henri Gillet
University of Illinois at Chicago

Derivations and descent in Characteristic p

Moshe Kamensky
University of Notre Dame

Differential tensor categories and model theory

I will suggest an axiomatisation of the categorical structure of the category Rep_G of representations of a linear differential algebraic group G. This is analogous to the description of Rep_G as a rigid abelian tensor category for a linear algebraic group G. I will present some constructions which suggest that one can do differential algebraic geometry within such a category.

Fedor Bogomolov
New York University

Closed holomorphic differentials and holomorphic webs

I will report on a progress in our joint work with Bruno de Oliveira. We call a symmetric holomorphic differential s on a complex manifold closed if it can be written as Πdf_i^{ni} in the local neighborhood of some point x_o of the manifold. This property holds then for a complementary of a finite set of divisors and hence such a differential defines a holomorphic web - a set of local codimension one foliations on the manifold (possibly singular). Note that any symmetric differential on surface defines a similar web. However the webs defined by closed differentials are very special and provide with restrictions on the topology of the ambient surface (or manifold). I will discuss some results and conjectures.

Ray Hoobler
CUNY City College

Applying Picard-Vessiot theory to Brauer groups

The delta-flat Grothendieck topology allows us to solve differential equations locally and use sheaf theory to obtain global results. This will be illustrated by showing that the differential Brauer group of a differential ring, constructed using differential Azumaya algebras, is the same as the Brauer group of the ring (http://arxiv.org/abs/1003.1421). Among topics for further discussion will be defining a Picard-Vessiot Grothendieck topology, what happens in the case of projective varieties, and applications suggested by others.

Dmitry Trushin
Moscow State University

Difference Nullstellensatz

We develop geometric theory of difference equations. One can show that difference fields are not suitable for our purpose. So, we have to seek for another class of difference rings. Instead of fields we consider the class of all simple absolutely flat difference rings and show how far we can get using them. As a demonstration, we show some applications to the Picard-Vessiot theory.

Bruno de Oliveira
University of Miami

Geometry of closed symmetric differentials of rank 1 and 2

This talk has a general theme the pursuit of a holomorphic theory of the fundamental group. Symmetric differentials on complex manifolds in general do not encode topological information. Is there a class of symmetric differentials (in all degrees) which encodes topological data? We propose that the class of what we call closed symmetric differentials is a good candidate. We will describe the geometric/topological properties associated with closed symmetric differentials of rank 1 and 2. As an example, we shall demonstrate that the presence of a closed symmetric differential of degree two on a complex surface X implies that either the fundamental group of X is infinite or there is a negative divisor E for which the complement X\E has an infinite fundamental group.