Logic I

MATH 711, TU/TH 4:15pm - 5:45pm, Rm. TBA

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The **objective** of this course is to introduce syntax and semantics of (finitary) first-order logic, also known as $L_{\omega\omega}$, and to study its fundamental properties and uses in various areas of mathematics. The presented material forms the basis for any of the classical subject areas of mathematical logic—set theory, recursion (or computability) theory, proof theory, and model theory.

First-order logic provides the language in which set theory, Peano arithmetic, and non-standard analysis are formulated, but it is as useful in the study of—among many other things—orderings, graphs, groups, rings, and fields. These latter are the types of structures I will concentrate on in the accompanying examples, and rudimentary knowledge of them is the only **prerequisite** for the course.

A fundamental result in the area and the first immediate goal of the course is the *Compactness Theorem*. Emphasizing the semantic aspect of the theory, I will prove this theorem using *ultraproducts*, a very useful tool in its own right. (Most classical texts take the route through the concept of *formal proof* and the corresponding *Completeness Theorem*, which in this course is deferred to the second semester that addresses the syntactic side of the theory in more depth.)

The course may be outlined as follows (and go beyond or deviate from that if time permits or interest suggests).

Structures: which is what it's all about
Languages: syntax
Semantics: truth and consequence, theories and axiomatizability, definability
Compactness: ultraproducts
Consequences: Löwenheim-Skolem Upward, non-standard models
Diagrams and Interpretations: Maltsev's local theorems of group theory
Orderings: Cantor's isomorphism theorem, ordinals and cardinals
Elementary equivalence: Löwenheim-Skolem Downward, Skolem's Paradox
Elimination of quantifiers: algebraically closed fields
Complete types (types of elements)

Literature: my own *Introduction to Model Theory* (Gordon & Breach 2000).

I'd be more than happy to answer any questions concerning the course and related literature via < *philipp.rothmaler@bcc.cuny.edu* >.