Indecomposability

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Outline of Topics

Foundational remarks regarding groups

Ranks, connectivity, and The Cassidy-Singer Problem

The linear almost simple case

The general solution, and a proof of the key lemma

Minchenko's Proof

Generalizations

Notation

- k will be a characteristic zero Δ-field.
- $\mathcal{M}$ will be a saturated enough model of $DCF_{0,m}$.
- C_δ is the definable subfield of $\mathcal{M}$.
- Generally, varieties, etc. will be over k
- I will abuse notation and equate varieties, definable sets, etc. with their $\mathcal{M}$-points. Please interrupt me if this (or anything) becomes unclear.

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An example

Let $\Delta = \{\delta_1, \delta_2\}$. Then consider the following group G of matrices of the form:

$$\begin{pmatrix} 1 & u_1 & u \\ 0 & 1 & u_2 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\delta_i(u_i) = 0$. Of course,

$$\begin{pmatrix} 1 & u_1 & u \\ 0 & 1 & u_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & v_1 & v \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & u_1 & u \\ 0 & 1 & u_2 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & v_1 & v \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$$
$$= \begin{pmatrix} 1 & 0 & u_1 v_2 - v_1 u_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A series of questions the example raises

- The derived subgroup is isomorphic $\mathbb{Q}(C_{\delta_1} \cup C_{\delta_2})$, where C_{δ_i} is the field of δ_i-constants. This is not a definable set.
- G was not "connected enough" to ensure that the derived subgroup was closed.
- We are generally interested in the problem of when a family of subvarieties of a differential algebraic group generates a differential algebraic subgroup.
- This phenomenon exists for general superstable groups; it is related to Cherlin's main conjecture. We will try to explain this at the end.

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Two categories

- **Δ-k-algebraic groups:** Let X be an abstract differential algebraic variety over k. That is, an object obtained by glueing together finitely many affine differential algebraic varieties U_i with differential rational transition maps f_{ij}. $X \times X \to X$ a Δ-morphism, that is, a map which is locally differential rational.
- Δ-k-definable groups: $X \subseteq \mathcal{M}^n$ is a definable set and $\cdot : X \times X \to X$ is a group operation whose graph is a definable set.

After a few more notes, we will explain why these two classes of groups are the same.

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Elimination of imaginaries

- Let T be any theory and $\mathcal{M} \models T$ saturated. If ϕ is a formula over $\mathcal{M}$, then B is a *canonical base* for ϕ if B is definably closed and whenever $\sigma \in Aut(\mathcal{M})$ fixes $\phi(\mathcal{M})$ as a set, $\sigma \in Aut(\mathcal{M}/B)$.
- A theory eliminates imaginaries if every formula has a canonical base.
- Suppose T eliminates imaginaries. Let E be a definable equivalence relation on $\mathcal{M}^n$. Then there is $m \in \mathbb{N}$ and a definable function $f : \mathcal{M}^n \to \mathcal{M}^m$ such that $E(\bar{x}, \bar{y})$ iff $f(\bar{x}) = f(\bar{y})$.

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An equivalence of categories

Any Δ-algebraic group defined over k can be canonically given the structure of a Δ-k-definable group:

- Fix an affine open covering of G given by $U_1, \dots, U_n$.
- Define H to be the *disjoint* union $\cup U_i$ quotiented by the k-definable equivalence relation E given by the transition functions f_{ij}. $f : G \cong H$
- By elimination of imaginaries, H is isomorphic to a Δ-k-definable group.

Pillay proved the other direction of the equivalence first under the assumption that $k \models DCF$ (1990). Later, this assumption was removed (1997).

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Types and definable groups

Differential algebraic groups are simply the definable groups in the theory DCF_m. For the rest of the talk, G is a k-definable group.

- DCF eliminates quantifiers (ie projections of constructible sets in the Kolchin topology are constructible).
- $p \in S(K) \leftrightarrow I_p = \{f | "f = 0" \in p\} \leftrightarrow V(I_p)$
  types $\leftrightarrow$ prime differential ideals $\leftrightarrow$ Irreducible Kolchin closed sets
- DCF_m is ω-stable
- There is a well-developed theory of ω-stable groups, which we will utilize, for instance, a key notion:

Definition

Let $p(x) \in S(K)$ be a complete type containing the formula $x \in G$. All of the complete types we deal with will contain this formula. Define

$$stab_G(p) = \{a \in G \mid \text{ if } b \models p,\ b \underset{K}{\perp\!\!\!\perp} a, \text{ then } ab \models p\}$$

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- Take $G = Z(x'')$ over an ordinary differential field k.
- Let p be the generic type of G. Then $stab_G(p) = G$.
- To emphasize, we are considering definable groups, and all ranks, generic types, etc. will be taken in that setting.

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Kolchin polynomials

Let Θ be the free commutative monoid generated by Δ. For $\theta \in \Theta$, if $\theta = \delta_1^{\alpha_1} \ldots \delta_m^{\alpha_m}$, then $ord(\theta) = \alpha_1 + \ldots + \ldots + \alpha_m$. The order gives a grading on the monoid Θ. We let

$$\Theta(s) = \{\theta \in \Theta : \ ord(\theta) \leq s\}$$

Theorem

Let $\eta = (\eta_1, \ldots, \eta_n)$ be a finite family of elements in some extension of k. There is a numerical polynomial $\omega_{\eta/k}(s)$ with the following properties.

1. *For sufficiently large $s \in \mathbb{N}$, the transcendence degree of $k((\theta\eta_j)_{\theta \in \Theta(s), 1 \leq j \leq n})$ over k is equal to $\omega_{\eta/k}(s)$.*
2. $deg(\omega_{\eta/k}(s)) \leq m$
3. $\omega_{\eta/k}(s) = \sum_{o \leq i \leq m} a_i \binom{s+i}{i}$. *In this case, a_m is the differential transcendence degree of $k\langle\eta\rangle$ over k.*

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Differential type and typical differential dimension

$$\omega_{\eta/k}(s) = \sum_{o \leq i \leq m} a_i \binom{s+i}{i}.$$

The degree of $\omega_{\eta/k}(s)$ is called the *differential type of η over k*. Notation: $\tau(\eta/k)$, where we often omit k if it is fixed by the context.

The leading coefficient is called the *typical differential dimension of η over k*. Notation: $a_\tau(\eta/k)$ or $a_\tau(\eta)$.

Let $\eta \models p \in S(k)$.

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Special groups

- The *strong identity component* $G_0 \lhd G$ is the smallest definable subgroup H of G such that $\tau(G/H) < \tau(G)$.
- G is *strongly connected* if $G = G_0$.
- G is *almost simple* if for any normal proper definable subgroup H of G we have $\tau(H) < \tau(G)$.
- Cassidy and Singer showed that every differential algebraic group has a composition series in which the successive quotients are almost simple. The quotients are unique up to permutation and isogeny.
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- The *strong identity component* $G_0 \lhd G$ is the smallest definable subgroup H of G such that $\tau(G/H) < \tau(G)$.
- G is *strongly connected* if $G = G_0$.
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- $\mathbb{G}_a$
- $\mathbb{G}_m$
- quasi simple algebraic groups restricted to definable subfields.
- Lets show $V(\delta_1 y - f(y))$ where $f \in K[\delta_2]$ is almost simple:
- For simplicity, let $\Delta = \{\delta_1, \delta_2\}$. A proper subgroup must be defined by a linear operator $g \in K[\Delta]$ such that $g(y) \notin \{\delta_1 y - f(y)\}$ in $K\{z\}$ and such that $H \subset \{y \in G \mid g(y) = 0\}$. We may assume that $g \in K[\delta_2]$. If g has order d, then for any $y \in H$, we have $k(y, \delta_2 y, \delta_2 y, \ldots) = k(y, \delta_2 y, \ldots, \delta_2^{d-1} y)$. So, H has type 0.

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- Let G Kolchin closure of the torsion points on a simple abelian variety A. It turns out that G is the kernel of a definable map $M : A \to \mathbb{G}_a$.
- For instance, take E_a be the elliptic curve with $y^2 = x(x-1)(x-a)$.
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$$M(x,y) = -\frac{y}{(x-a)^2} + \delta(2a(a-1)\frac{\delta(x)}{y}) + \frac{a(a-1)}{x-a}\frac{\delta(x)}{y}$$

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Lascar rank

Definition

Let p and q be types such that $p \subset q$. In this case, we say that q *is an extension* of p. We say that q is a *nonforking* extension of p if $\omega(q) = \omega(p)$.

Definition

Let p be a type. Then,

- $RU(p) \geq 0$ if p is consistent.
- $RU(p) \geq \beta$, where β is a limit just in case $RU(p) \geq \alpha$ for all $\alpha < \beta$.
- $RU(p) \geq \alpha + 1$ just in case there is a forking extension q of p such that $RU(q) \geq \alpha$.

For instance, if p is the generic type of $x'' = 0$ over k and q is the generic type of $x' = c$ over $k\langle c \rangle$, then q is a forking extension of p. $RU(p) = 2$, $RU(q) = 1$.

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Other connectedness notions

We say G is α-connected if $RU(G/H) \geq \omega^{\alpha}$ for all definable proper subgroups H of G.

Conjecture

For any type, $RU(p) \geq \omega^{\tau(p)}$.

Corollary

G is strongly connected iff G is $\tau(G)$-connected.

The conjecture comes down to finding chains of (uniformly defined families) of subvarieties of $loc(p)$.

The conjecture should be "hard" because the sort of varieties for which we are trying to find subvarieties behave geometrically like zero dimensional varieties. For instance, Pong showed that any generic hyperplane misses such a variety.

It seems to be false when working over specific (non differentially closed) fields.

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We understand the simple, definable groups:

Theorem

Let G be a simple differential algebraic group. Then G is definably isomorphic to the C' points of H, a simple algebraic group, where C' is a definable subfield.

The finite Morley rank version of the theorem is reasonably easy. In the ordinary case, there is a proof due to Buium using jet spaces. There does not seem to be a conceptually "easy" proof in the partial case in literature. Why not? Could Buium's proof be generalized?

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Lower Bound for Lascar rank

- Let G be a linear, almost simple DAG.
- It is easy to show that $G/Z(G)$ is simple, so Cassidy's theorem applies. The Lascar rank of G is a $\omega^{\tau}(G) \cdot dim(G/Z(G))$. This is easy, but Omar Léon Sanchez has a nice exposition of this on his webpage, which is likely to be helpful for understanding Lascar rank in DCF.
- So, G is $\tau(G)$-connected in the sense of Berline-Lascar. So, $[G, G]$ is definable.
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Zilber Indecomposability

A definable subset of $A \subseteq G$ is *indecomposable* if for every definable subgroup H of G, A is contained in a single coset or intersects infinitely many cosets.

Theorem

Let G be a group of finite Morley rank. Let $\{X_i : i \in I\}$ be a family of indecomposable subsets of G, each containing the identity. Then the subgroup generated by the family is definable and connected.

Of course, the theorem has generalizations to superstable and supersimple contexts.

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X_i is Indecomposable $\leftrightarrow$ for any $H \leq G$, X/H is either very large or $|X/H| = 1$.

"Very large" means of differential type $\tau(G)$.

Theorem

Let G be a differential algebraic group. Let $1 \in X_i$ for $i \in I$ be a family of indecomposable definable subsets of G. Then the X_i's generate an strongly connected differential algebraic subgroup of G.

The proof involves trying to control Kolchin polynomials - the key lemma (which was incorrect in the first versions of the paper!) has to do with canonical bases. I want to go through the corrected statement and proof here.

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The key lemma

Lemma

Suppose that $\tau(G) = n$. Suppose that $p(x) \in S(K)$ with "$x \in G$" $\in p(x)$ Then, suppose, for some finite $A \subseteq K$, that

$$\omega_{p|_A}(t) < \omega_p(t) + \binom{t+n}{n}$$

Then there is a tuple $\bar{c} \in K$ such that $\omega_p(t) = \omega_{p|_{\bar{c}}}(t)$ and $\omega_{\bar{c}/A}(t) < \binom{t+n}{n}$.

Let $\langle b_k \rangle_{k \in \mathbb{N}}$ be a Morley sequence over K in the type of p. By the characterization of forking in $DCF_{0,m}$ this simply means that for all $k \in \mathbb{N}$,

$$\omega_p(t) = \omega_{b_k/K}(t) = \omega_{b_k/K \cup \{b_0, \ldots, b_{k-1}\}}(t)$$

We do not know, however, that the same holds over the (arbitrary) subset $A \subseteq K$.

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Let $\langle b_k \rangle_{k \in \mathbb{N}}$ be a Morley sequence over K in the type of p. By the characterization of forking in $DCF_{0,m}$ this simply means that for all $k \in \mathbb{N}$,

$$\omega_p(t) = \omega_{b_k/K}(t) = \omega_{b_k/K \cup \{b_0, \dots, b_{k-1}\}}(t)$$

We do not know, however, that the same holds over the (arbitrary) subset $A \subseteq K$.

The key lemma

Lemma

Suppose that $\tau(G) = n$. Suppose that $p(x) \in S(K)$ with "$x \in G$" $\in p(x)$ Then, suppose, for some finite $A \subseteq K$, that

$$\omega_{p|_A}(t) < \omega_p(t) + \binom{t+n}{n}$$

Then there is a tuple $\bar{c} \in K$ such that $\omega_p(t) = \omega_{p|_{\bar{c}}}(t)$ and $\omega_{\bar{c}/A}(t) < \binom{t+n}{n}$.

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We do not know, however, that the same holds over the (arbitrary) subset $A \subseteq K$.

- The sequence is still necessarily A-indiscernible, that is $tp(b_k/A)$ does not depend on k. It is not necessarily A-independent. In general, we simply know that $\omega_{b_k/A\cup\{b_0,\dots,b_{k-1}\}}$ is a decreasing sequence of polynomials, again, ordered by eventual domination. Kolchin polynomials are well-ordered by eventual domination (Sit).
- Fix k such that if $n \geq k$, the sequence is constant. That is, above k, we know that we have a Morley sequence over $A \cup \{b_0, \dots, b_{k-1}\}$ in the type of p.
- Now, fix a model $K' \models DCF_{0,m}$ with K' containing K and $\{b_0, \dots, b_{k-1}\}$. Take p' the nonforking extension of p to K'.
- We can get elements $\bar{c} \subseteq acl(A \cup \{b_0, \dots, b_{k-1}\})$ such that p' does not fork over $\bar{c}$.
- We know that

$$\omega_{p|_A}(t) = f(t) + h(t)$$

where

$$f(t) = \sum_{i=n+1}^{m} c_i \binom{t+i}{i}$$

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- By assumption, $\omega_{p|_A}(t) < \omega_p(t) + \binom{t+n}{n}$. Thus, $f(t) \leq \omega_p(t)$.
- $\langle b_i \rangle$ was an indiscernible sequence, so if we define $\bar{b} := (b_0, \ldots, b_{k-1})$, then

$$k \cdot f(t) \leq \omega_{\bar{b}/K}(t)$$

- So,

$$k \cdot f(t) \leq \omega_{\bar{b}/A \cup \bar{c}}(t) \quad (1)$$

- Now, for $i = 0, 1, \ldots, k-1$, we have that

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Theorem

(Cassidy) Every simple linear differential algebraic group is isomorphic to the F'-points of an algebraic group where F' is a definable subfield.

Theorem

Every noncommutative almost simple differential algebraic group is isomorphic to a linear differential algebraic group. Suppose that G is strongly connected. Every definable normal subgroup, N, with $\tau(N) < \tau(G)$ is central. Let G be almost simple. Then $G/Z(G)$ is simple.

So, noncommutative almost simple groups are perfect central extensions of algebraic groups. By applying some results of Steinberg, along with the structure theory of differential algebraic groups, one can show $Z(G)$ is actually finite (Minchenko, Altinel-Cherlin in the FMR case).

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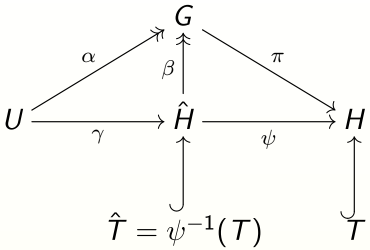
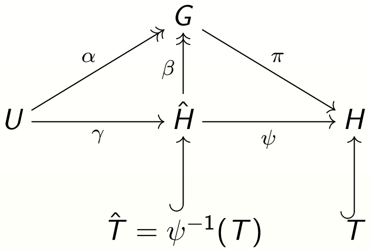


where $\hat{H}$ is the universal Chevellay group covering H. U is the universal central extension of algebraic groups. Perfection of G gives that α is onto. The rest of the arguement requires some structure theory from differential algebraic groups.

Generalizations?

ω-stable, Superstable, Supersimple, Difference, Difference-differential, D-groups, $\mathcal{E}$-groups in the sense of Moosa-Scanlon, ...



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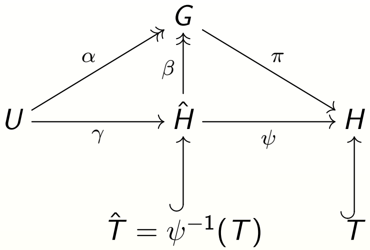

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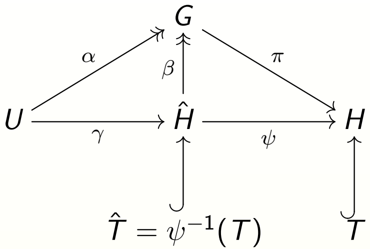

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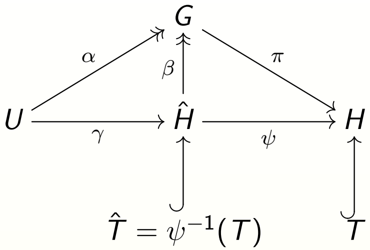

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Thanks for listening

Thanks to D. Marker for many useful conversations about this work.

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Thanks very much for listening.